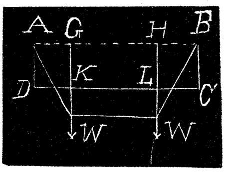

weight of each bead; (x, y) the coördinates of F. Then we have $\surd(HB^2+HF^2)+\frac{1}{2}EF=a$.

$\therefore \surd[(b-x)^2+(a-b+y)^2]+x=a$,

or $y^2+2(a-b)y+2(a-b)x=2b(a-b)$,

or $(y+a-b)^2+2(a-b)x=(a-b)(a+b)=a^2-b^2$.

$\therefore (y+a-b)^2=(a-b)(a+b-2x)$, this is the locus of the bead F. $\therefore$ Each bead describes the arc of a parabola. Let T=tension of the string at any point, $\theta=\angle HFB$; resolving vertically, $\frac{1}{2}T\cos\theta=W$, or $T=2W/\cos\theta$. Now $1/\cos\theta=BF/HF=\surd[(b-x)^2+(a-b+y)^2]/(a-b+y)$.

$\therefore 1/\cos\theta=(a-x)/(a-b+y)$. $\therefore T=2W(a-x)/(a-b+y)$.

When $x=b$, $y=0$, and $T=2W$.

$\therefore x=0$, $y=\surd(a^2-b^2)-(a-b)$, $T=2aW/\surd(a^2-b^2)$.

DIOPHANTINE ANALYSIS.

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104. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

(1). The cube root of three cube numbers equals the square root of two square numbers. Determine the numbers.

(2). The sum of the square roots of three square numbers equals the sum of the cube roots of three cube numbers. Determine the numbers.

Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

If we assume the three cube numbers to be x^3, y^3, and z^3, and the two square numbers to be u^2 and v^2, we are to find values of x, y, z, u, and v so that the equation $x+y+z=u+v$ shall be satisfied. Any four of these values may be selected at pleasure and the fifth one may then be determined. For example, let $z=1$, $y=2$, $u=3$, $v=4$. Then $x=4$. The cube numbers are 64, 8, 1, and the two square numbers are 9 and 16. By assuming any values whatever for y and z, and any values of u and v such that their sum is greater than the sum of y and z, as many positive numbers may be found satisfying the conditions of the problem as may be desired. The second part of the problem may be solved in the same way.

105. Proposed by HARRY S. VANDIVER, Bala, Pa.

Every odd factor of a^n+b^n is of the form $1(\text{mod}2n)$.

No solution of this problem has been received.

106. Proposed by L. C. WALKER, A. M., Graduate Student. Leland Stanford Jr. University, Cal.

There is a series of rational triangles whose sides have a common difference of unity. Calling the one whose sides are 3, 4, 5 the first triangle, find the sides of the next five triangles, and a general expression for the sides of the nth triangle.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; CHAS. C. CROSS, Memphis, Tenn. and J. E. SANDERS, Hackney, Ohio.

Let $x-1$, x, and $x+1$=the sides of a rational triangle.

Then, $(Area)^2=\dfrac{3x}{2}\cdot\dfrac{x}{2}\cdot\dfrac{x+2}{2}\cdot\dfrac{x-2}{2}=\dfrac{3x^2(x^2-4)}{16}=\square$.

$\therefore 3(x^2-4)=\square=9m^2$; and $x=\sqrt{(3m^2+4)}$.

To rationalize $\sqrt{(3m^2+4)}$, we use the convergents of the $\sqrt{3}=\frac{1}{1}, \frac{2}{1}, \frac{5}{3}, \frac{7}{4}, \frac{19}{11}, \frac{26}{15}, \frac{71}{41}, \frac{97}{56}, \frac{265}{153}, \frac{362}{209}, \frac{989}{571}, \frac{1351}{780}$, etc. Beginning with $\frac{2}{1}$ and taking the alternate convergents, we put x=twice the numerators and m=twice the denominators, and obtain the following set of pairs of values:

x=4, 14, 52, 194, 724, 2702, etc.
m=2, 8, 30, 112, 418, 1560, etc.

$\therefore$ The sides of the first six triangles are 3, 4, 5; 13, 14, 15; 51, 52, 53; 193, 194, 195; 723, 724, 725; 2701, 2702, 2703. The area$=3mx/4$.

Taking x_{n-2} and x_{n-1} as any two consecutive middle sides, we find the next middle side, or $x_n=4x_{n-1}-x_{n-2}$. This law of formation gives the following general expression for the middle side of the nth triangle, using 4 and 2 (=middle side of "straight line" triangle) as the middle sides of two consecutive triangles:

$$x_n=4\Big[4^{n-1}-(n-2)4^{n-3}+\frac{(n-3)(n-4)}{2}4^{n-5}-\frac{(n-4)(n-5)(n-6)}{2\times3}4^{n-7}+$$

$$\frac{(n-5)(n-6)(n-7)(n-8)}{2\times3\times4}4^{n-9}-\ldots\Big]-2\Big[4^{n-2}-(n-3)4^{n-4}+\frac{(n-4)(n-5)}{2}4^{n-6}$$

$$-\frac{(n-5)(n-6)(n-7)}{2\times3}4^{n-8}+\frac{(n-6)(n-7)(n-8)(n-9)}{2\times3\times4}4^{n-10}-\ldots\Big].$$

In finding values, we use those terms only which, in the substitutions for n, have *positive* and *zero* exponents.

Take $n=5$; then $x_5=4[4^4-3\times4^2+4^0]-2[4^3-2\times4']=724$.

Excellent solutions were received from *G. B. M. ZERR*, *J. SCHEFFER*, and the late *J. H. DRUMMOND*.

AVERAGE AND PROBABILITY.

128. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Two small circles are drawn on the surface of a sphere so as to intersect; find average area of the spherical triangle formed by joining the poles and one of the intersections of the small circles with arcs of great circles.